Monarch matrix math

Monarchs live and reproduce in the northern and eastern United States during the summer months, then migrate to central Mexico to live in the mountains over the winter. The summer butterflies reproduce every *** weeks on average, while the butterflies wintering in Mexico don't reproduce all winter, although they might reproduce once in April before flying north. This time without reproduction is called "reproductive diapause."

- 1. Draw a life cycle graph for the monarch butterfly. The **vertices** of the graph should be labeled as follows:
 - 1. The immature stage: egg, larval, and pupal development.
 - 2. The first month of life for butterflies in reproductive diapause.
 - 3. The second and later months of life for butterflies in reproductive diapause.
 - 4. The first month of life for butterflies in the north, who are reproductively active.
 - 5. The second and later months of life for butterflies in the north who are reproductively active.

Draw the **arrows** of the graph as solid arrows if the arrows represent transitions made by butterflies that are reproductively active and as dotted arrow if the arrows represent transitions made by butterflies in reproductive diapause (or the southern butterflies who become reproductively active only in April).

Guiding questions: Which groups can have arrows pointing back at themselves, and which can't? What arrows should point back at group (1), the immature butterflies?

2. Label the arrows on your graph with the appropriate labels: s_L for survival of immature butterflies, s_{ow}^1 for the overwintering survival rate of 1-month-old butterflies, s_{ow}^2 for the overwintering survival rate of 2-month old butterflies, $s_b^!$ for the survival rate of breeding 1-month-old butterflies, s_b^2 for the survival rate of breeding 2-month-old butterflies, and then f_b^1 and f_b^2 for fecundity of breeding 1- and 2-month-old butterflies. Are you missing anything?

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3. Create a 5×5 matrix representing what happens to summer butterflies (the ones who reproduce and die up in the north). These should correspond to your solid arrows.

4. Create a 5×5 matrix representing what happens to overwintering butterflies (the ones who are in reproductive diapause all winter). These should correspond to your dashed arrows.

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5. If you have a vector representing the butterfly population in Minnesota in July of 2010, how should you compose matrix multiplications to get the butterfly population in Michoacan in January of 2012?