Modeling Tides: Derivatives

This worksheet will guide you through modeling tidal water levels and their rates of change outside Point Reyes, California, using a spreadsheet program like Excel or Google Drive. Every spreadsheet program is a little different, but you can use almost any spreadsheet program to carry out these operations!

Tides are complicated and depend on many factors including the moon's gravitational pull. Tides can be roughly modeled by sine and cosine waves, though: functions of the form

$$f(t) = A\cos(Bt) + C.$$

We use t to represent time and $f(t)$ represents water level.

1. Start with the provided spreadsheet, which has time in column one and actual water level data from Point Reyes, California, in column two.

2. Find an equation for water level using the following steps:

   (a) Data about the tides in Point Reyes tell us that the water height ranges between 1.4 feet and 6 feet. What is the *average water level*?

   (b) Since you know the lowest water level is 1.4 feet and the highest is 6 feet, what is the *amplitude* of the cosine wave?

   (c) On October 4, 2013, the water was at high tide at 6 am and 6 pm. The water was at low tide at noon and at midnight. What is the *period* of the water height function?

   (d) Using the previous two parts, figure out A and B.

   (e) Write the equation for the model you have created!

   $$f(t) = \hspace{6cm} .$$

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3. Go back to the spreadsheet and fill in the third column with your predicted data. In the first free cell, you will enter your formula by typing the $=$ sign first and then the rest of the formula. Once you've pressed enter on that formula cell, you should get a number. Now copy and paste to the rest of the cells in that second column. You may need to "paste special."

4. The fourth column is for rate of change calculated from the actual data from Point Reyes. Find a formula for the rate of change of water level over one hour. Use that formula to fill in column four, labelled with "Actual Rate of Change."

5. Fill the fifth column of the spreadsheet with the rate of change of water level that your model predicts at each time, by using the derivative of your formula $f(t)$.

6. Highlight all five columns and make a chart out of the information, graphing your prediction and its derivative against the real data.

7. How good is your model for water level? Evaluate its strengths and weaknesses:

8. How good is your model for rate of change of water level?

9. Can you find a better model for either water level and rate of change?

10. Extra: find more data on the tides at Point Reyes from the National Oceanic Administration's Tides and Currents website and build a better model!