Maximum sweetness

Nitrogen is a hugely important nutrient for plants, and sugar beets are no exception. Farmers cannot control temperature, wind, rain, or snow, but they can control nitrogen levels in soil. Too much nitrogen, besides being bad for the environment, increases impurities in the sugar beets and reduced sucrose levels. Too little leads to poor leaf canopies and poor growth. Some nitrogen is always present in our soil, so how much nitrogen should farmers apply?

In "Nitrogen Fertilization of Sugarbeets in West-Central Minnesota 1976-1981," Malzer and Buzicky proposed several equations for recoverable sugar (RS) yields:

$$RS = 4155 + 37.2x - 0.2069x^2 \quad (1)$$

$$RS = 3733 + 37.7y - 0.19546x^2 - 6.1(.3z)^2 \quad (2)$$

$$RS = 4225 + 20.3(x + f) - 0.0655(x + f)^2 \quad (3)$$

Here, x represents nitrate-nitrogen level between 0 and 2 feet, y nitrate-nitrogen level between 0 and 3 feet, and z nitrate-nitrogen levels between 2 and 3 feet. The variable f is for fertilizer nitrogen. All units are in pounds per acre (lb/A).

1. Equation 1 looks at yields without fertilization. What nitrate-nitrogen level in soil between 0 and 2 feet deep produces the best recoverable sugar yield?

2. What is the maximum RS yield in the situation described by situation (1)?

3. Equation 3 looks at yields with fertilization. Treat $x + f$ as a single variable and find what value of $x + f$ produces the best recoverable sugar yield.

4. How much recoverable sugar would you get, at best, according to Equation 3?

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5. What is the difference between best yield in the fertilized and non-fertilized situations?

6. Equation 2 seems to take into account *three* variables. How can you relate the variables so that you can transform this into a two-variable equation?

7. In single-variable calculus, we can still analyze some two-variable equations. After finding RS in terms of x and z, plot graphs for RS depending on x for $z = 30, 35,$ and 40. Which value of x is best for each of these values of z? Can you find the values of x and z that give the absolute best RS yield through numerical experimentation?

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Using the first and second derivative tests

8. Using the first derivative test, *prove* that your answers to questions 1 and 2 are optimal. Use a full English sentence to explain your result.

9. Using the second derivative test, *prove* that your answers to questions 3 and 4 are optimal. Use a full English sentence to explain your result.

10. Can you use the Extreme Value Theorem and its corollaries to prove that your results are optimal? Why or why not? Write your reasoning in full sentences.