

Naproxen in water

Naproxen, a pain reliever used for everything from headaches to cramps, is found in many waterways these days, especially downstream from wastewater treatment plants. This worksheet uses information from a paper¹ which examines naproxen in the Trinity River in Dallas, Texas.

Photolysis – the breakdown of molecules due to exposure to sunlight – is an important cause of decay of naproxen in waterways. The amount of sunlight that reaches particles in waterways depends on how clear the water is (turbidity, described by a parameter α) and how deep the particles are (z meters).

In the photic zone, the depth which sunlight can reach, the *rate of decay* of naproxen is

$$k_{phot}(z) = k_{surf} \frac{1 - e^{-\alpha z}}{\alpha z}.$$

1. k_{surf} is always positive. Is $k_{phot}(z)$ an increasing or a decreasing function? Show your answer is true using mathematics and check with your intuition.

2. The authors of the paper found $k_{surf} = 0.39 \text{ day}^{-1}$ for the Trinity River. How fast is $k_{phot}(z)$ changing 20 centimeters below the surface?

depth z in meters	$k'_{phot}(0.2)$
k_{phot} for muddy river: $\alpha = 6 \text{ m}^{-1}$	
k_{phot} for medium river: $\alpha = 3 \text{ m}^{-1}$	
k_{phot} for clear river: $\alpha = 1 \text{ m}^{-1}$	

¹Attenuation of Wastewater-Derived Contaminants in an Effluent-Dominated River, Fono, Kolodziej, and Sedlak, Environmental Science & Technology 2006 40 (23), 7257-7262

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3. Use the table on the previous page (and some other calculations) to fill in the following sentence. Circle one of increasing or decreasing.

In a muddy river, the rate of photolysis k_{phot} for naproxen 20 centimeters below the surface of the water is _____ day^{-1} , and that rate is increasing/decreasing at _____ per meter.

Change turbidity, not depth!

4. It seems that naproxen doesn't settle like sand does, even with chemical help². After a heavy rain, sediment-filled runoff will stream into rivers and increase turbidity. Let's look at $k_{phot}(\alpha)$, keeping z constant. For naproxen at 20 centimeters below the surface of the Trinity River, what is the average change in k_{phot} as α changes from 6 m^{-1} to 1 m^{-1} ?

5. What is the instantaneous rate of change with respect to α at $\alpha = 3 \text{ m}^{-1}$, if z stays constant at 20 centimeters below the surface?

²"Occurrence of pharmaceuticals in river water and their elimination in a pilot-scale drinking water treatment plant," Vieno, Härkki, Tuhkanen, and Kronberg, Environmental Science & Technology 2007 41 (14), 5077-5084