Naproxen in water

Naproxen, a pain reliever used for everything from headaches to cramps, is found in many waterways these days, especially downstream from wastewater treatment plants. This worksheet uses information from "Attenuation of Wastewater-Derived Contaminants in an Effluent-Dominated River," published in 2006, which examines naproxen in the Trinity River in Dallas, Texas.

Photolysis – the breakdown of molecules due to exposure to sunlight – is an important cause of decay of naproxen in waterways. The amount of sunlight that reaches particles in waterways depends on how clear the water is (described by a parameter α) and how deep the particles are (z meters).

In the photic zone, the depth which sunlight can reach, the *rate of decay* of naproxen is

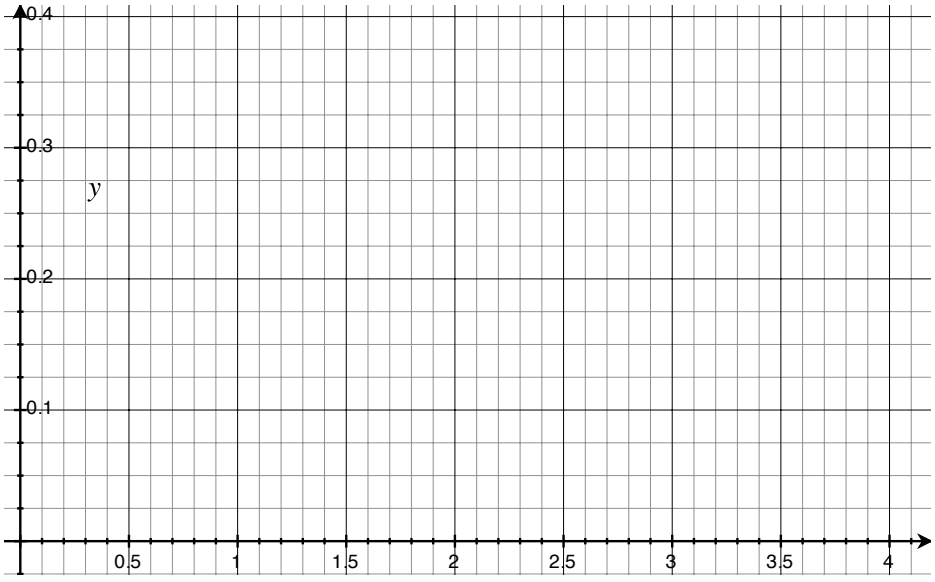
$$k_{phot}(z) = k_{surf}\frac{1-e^{-\alpha z}}{\alpha z}.$$

1. Without knowing k_{surf} or α, what are the domain and range of $k_{phot}(z)$?

2. The authors of the paper found $k_{surf} = 0.39\ \text{day}^{-1}$ for the Trinity River. Use this to fill in the table below:

depth z in meters	1	2	3	4
k_{phot} for muddy river: $\alpha = 6\ m^{-1}$				
k_{phot} for medium river: $\alpha = 3\ m^{-1}$				
k_{phot} for clear river: $\alpha = 1\ m^{-1}$				

3. Use your table to graph $k_{phot}(z)$ below for the three values of α given above.



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4. Calculate values of k_{phot} very near $z = 0$ by using $z = 0.1,\ z = 0.01,\ z = 0.001$ and $\alpha = 6$. Use these numerical approximations to compute $\lim_{z \to 0} k_{phot}(z)$. Does this fit with your physical understanding? Do you know how to compute this limit using limit rules? Discuss!

5. Calculate $\lim_{z \to \infty} k_{phot}(z)$. At the same time, ask yourself how many infinitely deep rivers you've encountered.

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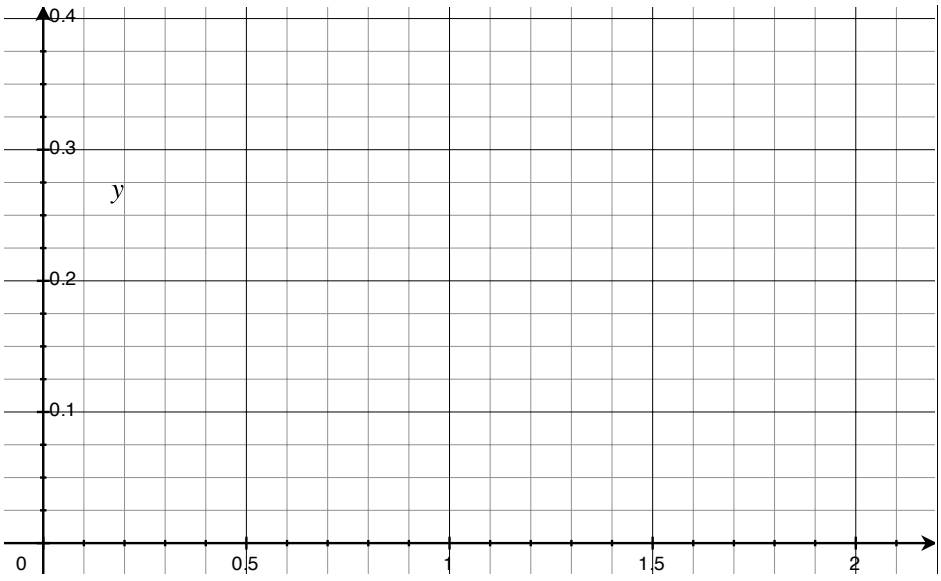
6. Scientists need to determine the depth of the photic zone when they research attenuation of pharmaceuticals. (Why would this matter?) For the Trinity River, the depth of the photic zone z_{phot} was measured to be 22 cm. For depths greater than 22 cm, the paper authors use the formula

$$k_{phot\ (deep)}(z) = \frac{k_{phot} z_{photic}}{z}, \quad z > z_{phot}.$$

Using $\alpha = 6\ m^{-1}$ and $k_{surf} = 0.39\ \mathrm{day}^{-1}$, write a formula for the rate of decay of naproxen as a piecewise function. (Watch your units!)

7. Graph the piecewise function $k_{phot}(z)$ you just discovered on the following axes:



8. What is $\lim_{z \to z_{phot}} k_{phot}(z)$? Is the function k_{phot} continuous at z_{phot}?